



DCS/CSCI 2350: Social & Economic Networks

*What can we say about a
network having friends as well as
enemies?*

Structural Balance

Reading: Ch 5 of EK

Mohammad T. Irfan

World War I

The New York Times

The Opinion Pages

Opinionator

OPINIONATOR

The Enemy of My Enemy

By STEVEN STROGATZ FEBRUARY 14, 2010 5:30 PM

It's traditional to teach kids subtraction right after addition. That makes sense — the same facts about numbers get used in both, though in reverse. And the black art of “borrowing,” so crucial to successful subtraction, is only a little more baroque than that of “carrying,” its counterpart for addition. If you can cope with calculating $23 + 9$, you'll be ready for $23 - 9$ soon enough.

At a deeper level, however, subtraction raises a much more disturbing issue, one that never arises with addition. Subtraction can generate negative numbers. If I try to take 6 cookies away from you but you only have 2, I can't do it — except in my mind, where you now have negative 4 cookies, whatever that means.

Subtraction forces us to expand our conception of what numbers are. Negative numbers are a lot more abstract than positive numbers — you can't see negative 4 cookies and certainly can't eat them — but you can think about them, and you *have* to, in all aspects of daily life, from debts and overdrafts to contending with freezing temperatures and parking garages.

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Steven Strogatz is the Schurman Professor of applied mathematics at Cornell University. Among his honors are MIT's highest teaching prize, membership in the American Academy of Arts and Sciences, and a lifetime achievement award for communication of math to the general public, awarded by the four major American mathematical societies. A frequent guest on National Public Radio's "Radiolab," he is the author, most recently, of "The Joy of x," which grew out of his previous Opinionator series "The Elements of Math." He lives with his wife and two daughters in Ithaca, N.Y. Follow him on Twitter @stevengstrogatz

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THE STONE



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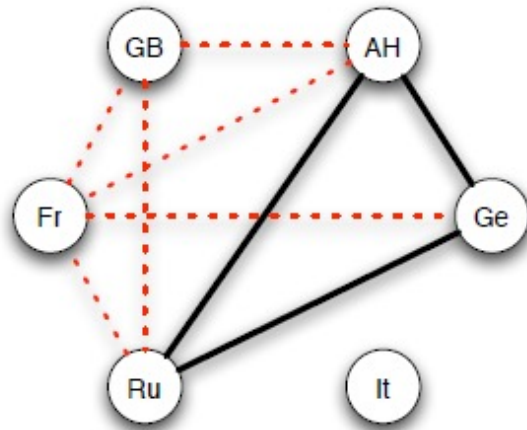
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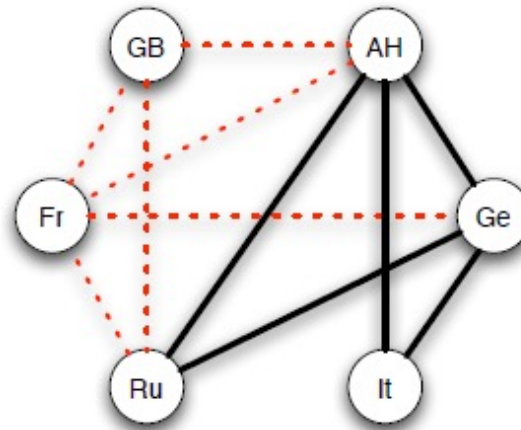
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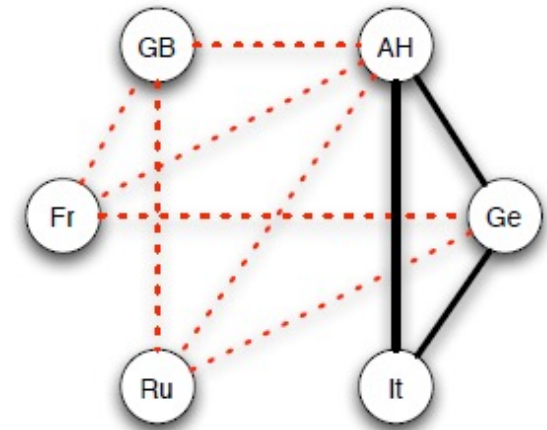
Evolution of international relationship



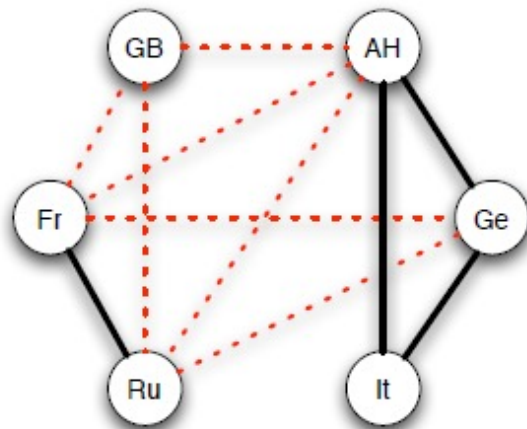
(a) *Three Emperors' League 1872-81*



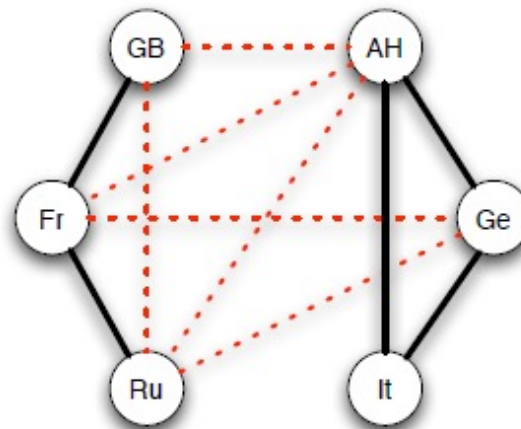
(b) *Triple Alliance 1882*



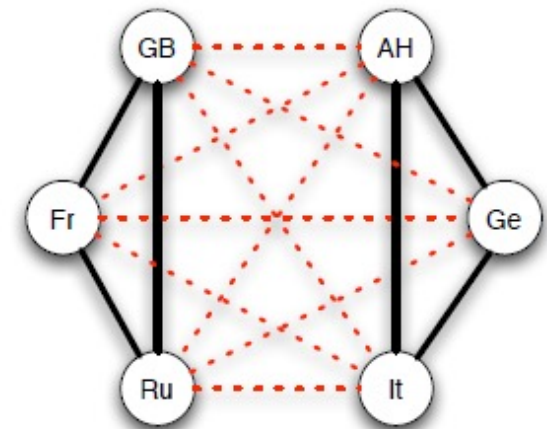
(c) *German-Russian Lapse 1890*



(d) *French-Russian Alliance 1891-94*



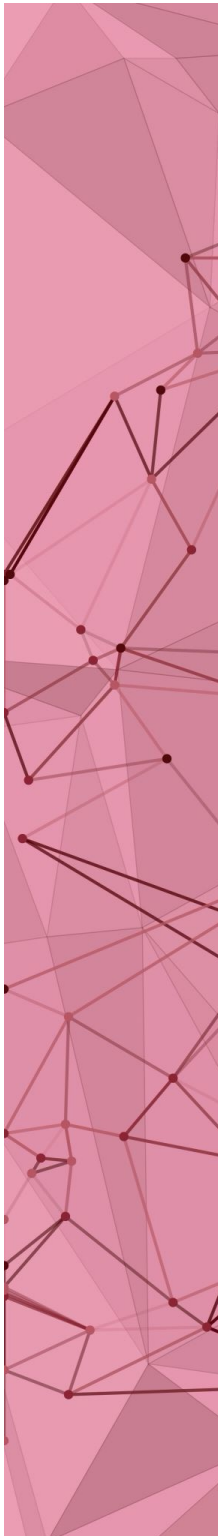
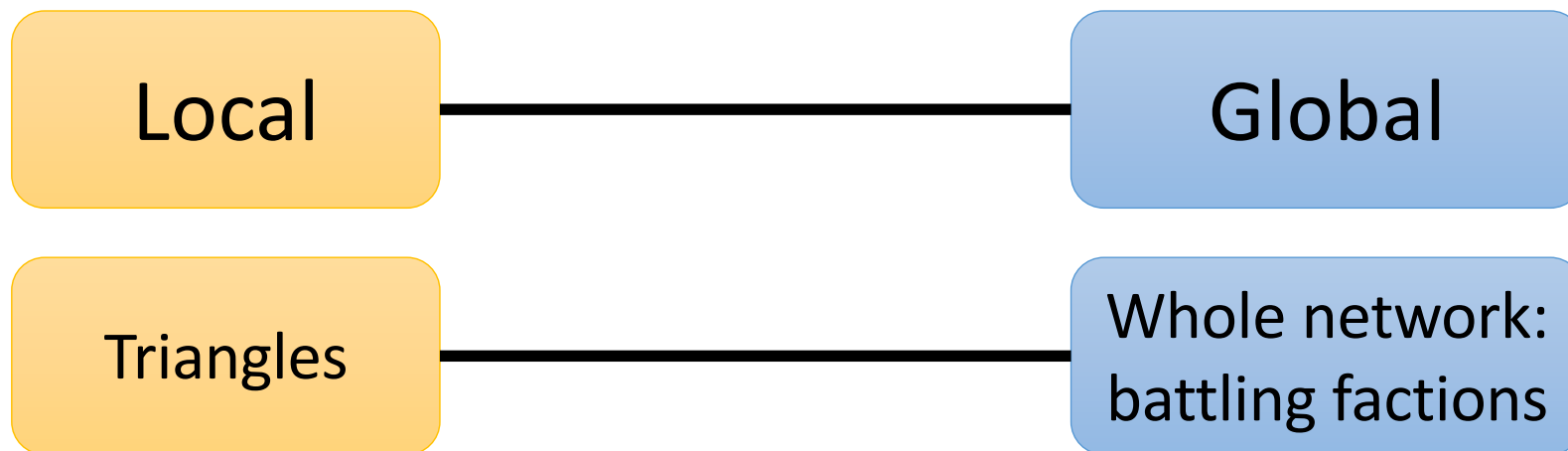
(e) *Entente Cordiale 1904*



(f) *British Russian Alliance 1907*

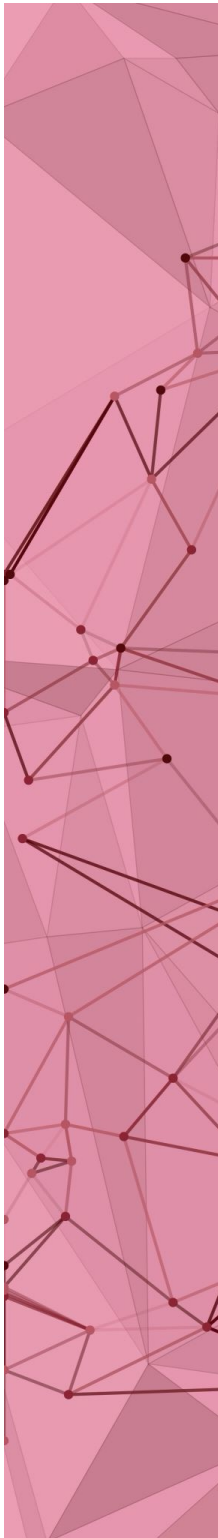
Agenda

- Network of friends and enemies
- Balanced/stable configurations



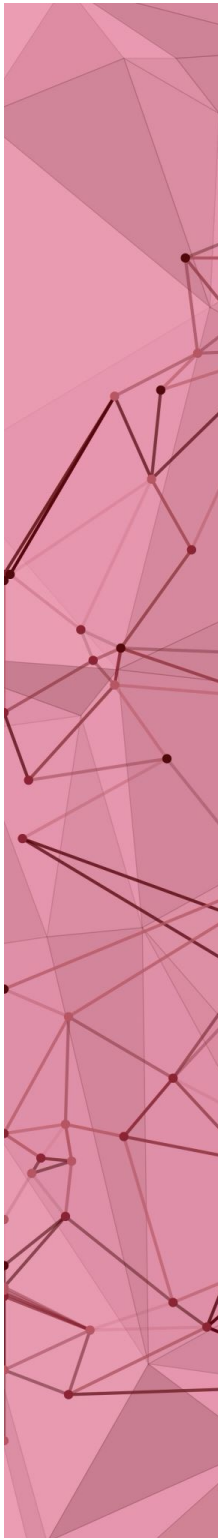
Founders

- Psychologist Fritz Heider (1940s)
- Mathematicians Cartwright and Harary (1950s)

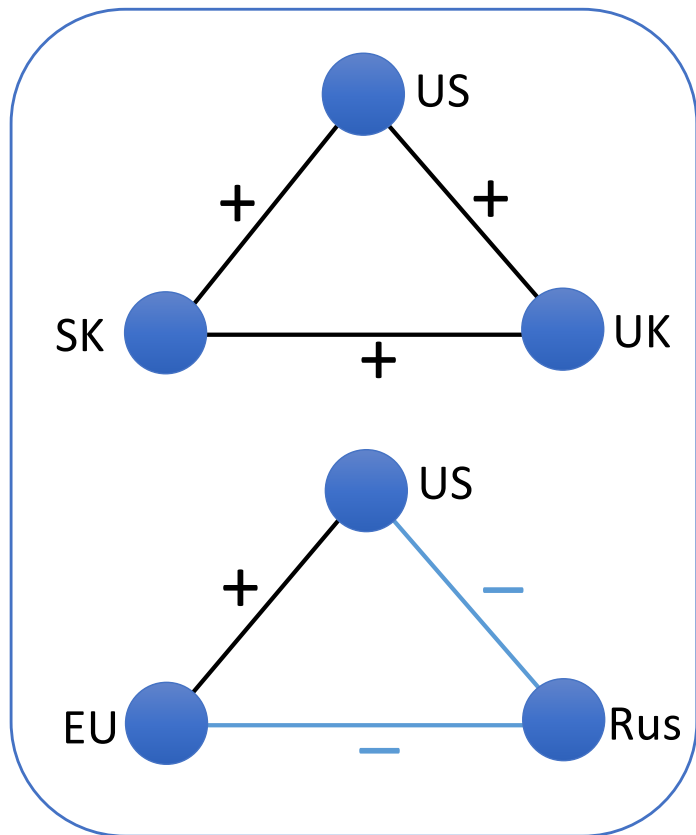


First model (to be generalized later)

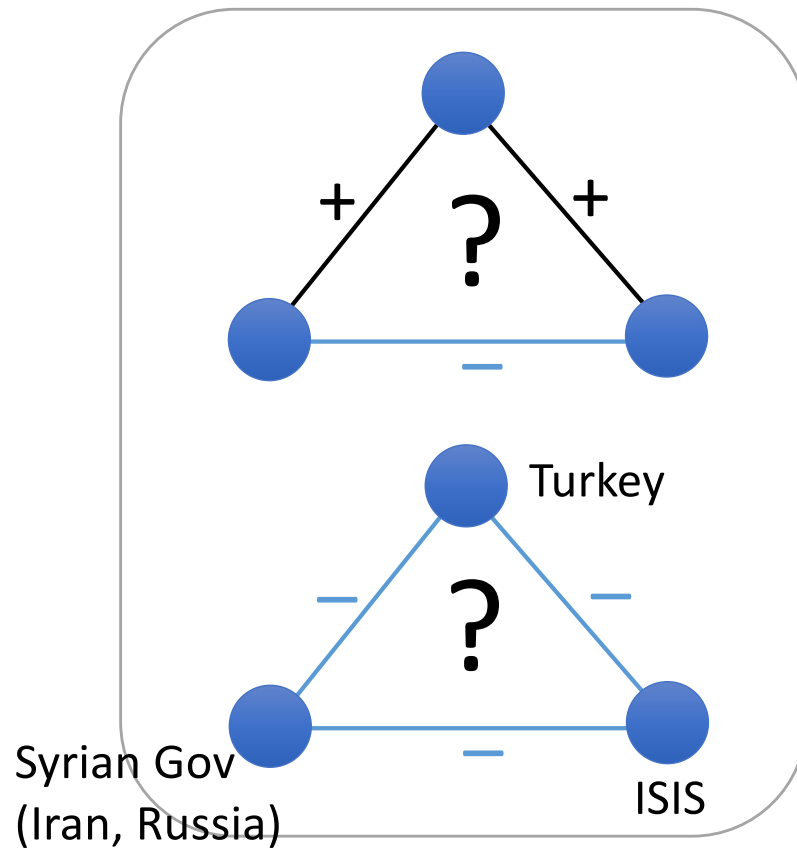
- **Complete graph** (edge between each pair of nodes)
- Each edge: either + or –
- Example: sports team, countries, etc.



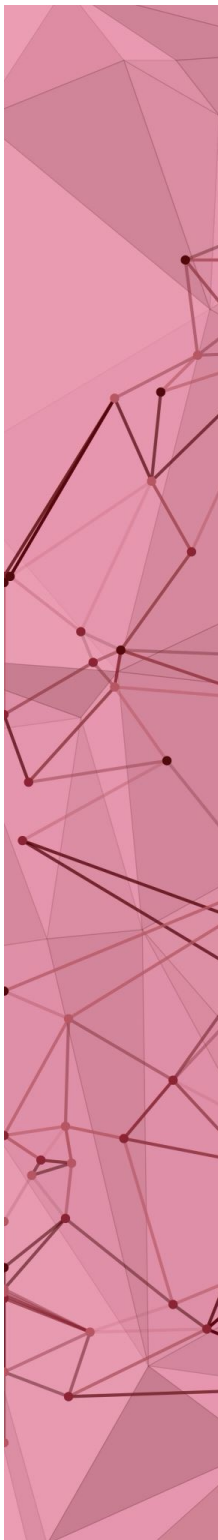
Structural balance



Balanced



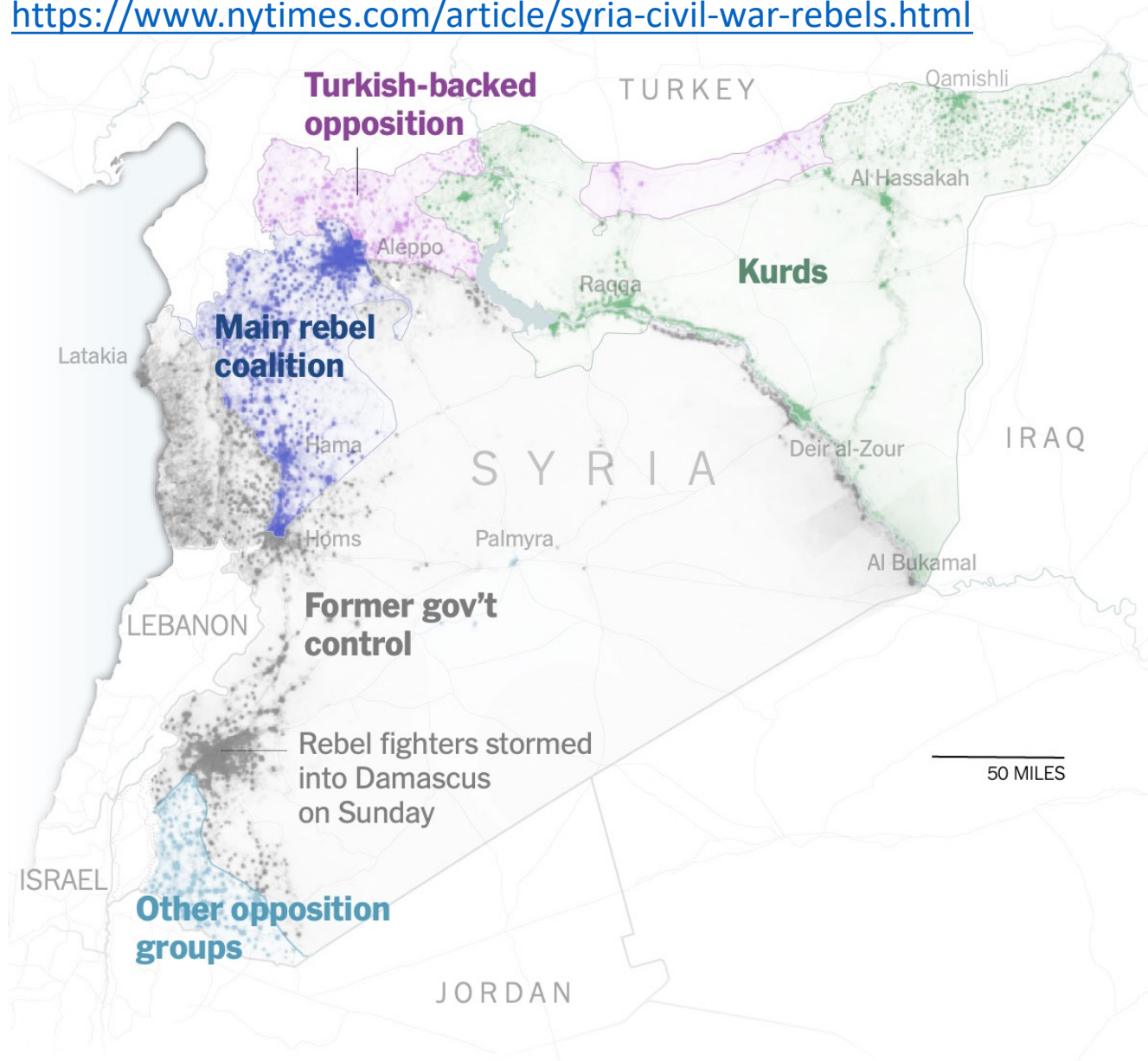
Unbalanced



What is the situation on the ground?

Dec 9, 2024

<https://www.nytimes.com/article/syria-civil-war-rebels.html>



Note: Areas of control are as of 10 p.m. local time on Saturday night, before rebels stormed into Damascus Sunday morning. The main rebel coalition is led by Hayat Tahrir al-Sham. • Sources: Institute for the Study of War and AEI's Critical Threats Project • By Samuel Granados

Assad's Downfall Marks a New Realignment in the Middle East

Turkey gains, Russia and Iran lose sway and Syrians hope a new round of civil war can be averted

By [Yaroslav Trofimov](#) [Follow](#)

Dec. 8, 2024 at 12:19 pm ET

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DOHA, Qatar—The struggle to depose Bashar al-Assad's regime took more than 13 years, causing untold death and destruction, because outside powers turned Syria into a proxy battlefield for their own ambitions.

When Assad's government [finally fell on Sunday morning](#), after a lightning campaign that took his friends and foes by surprise, it was [toppled by the Syrians themselves](#)—as the regime's main benefactors were enfeebled, and distracted, by mayhem elsewhere.

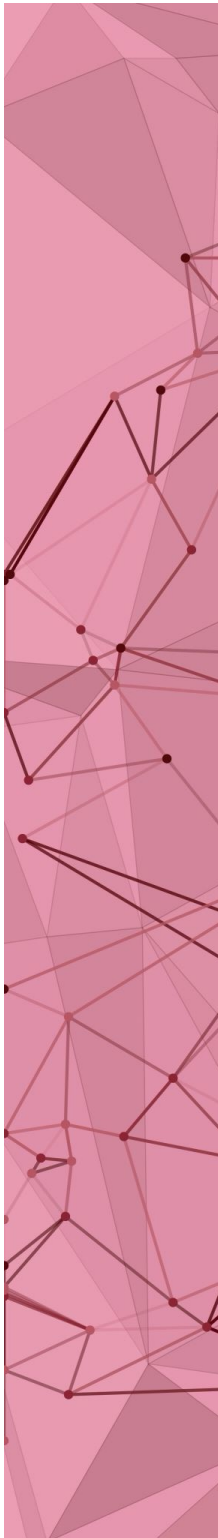
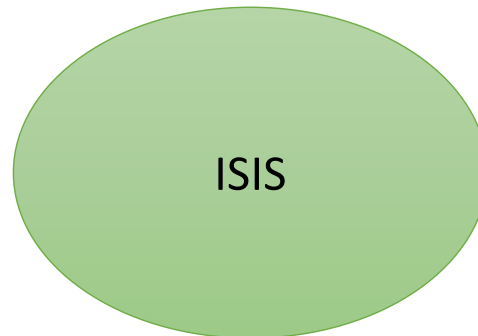
The outcome of this tectonic shift is bound to have a profound impact on the Middle East, and on the global balance of power.

Much will depend on how orderly the transition to a new rebel-led administration will be, and to what extent the rival Syrian factions—including the Kurdish and Alawite minorities—will be able to avoid further conflict.

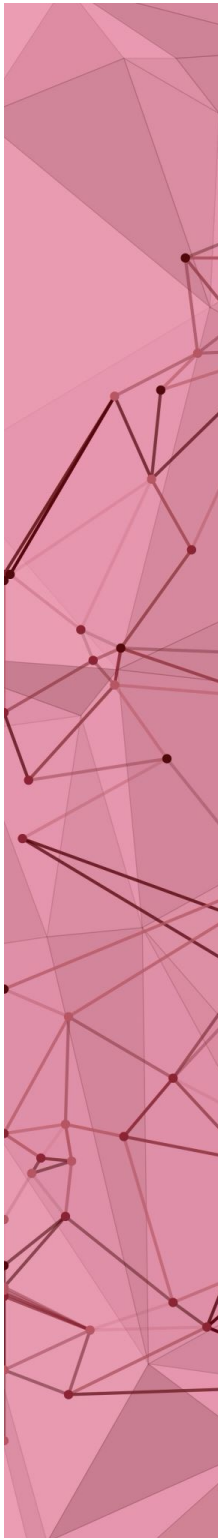
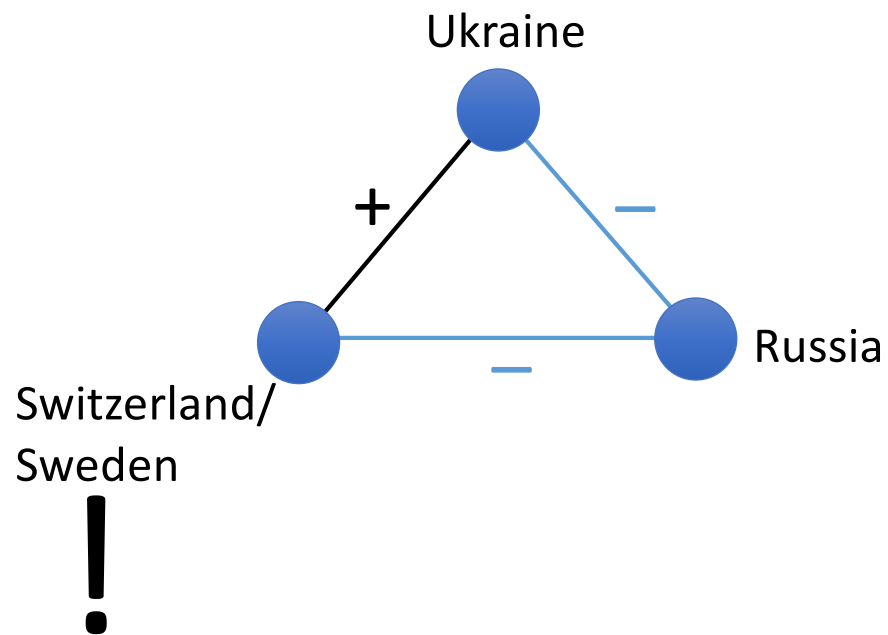
<https://www.wsj.com/world/middle-east/assads-downfall-marks-a-new-realignment-in-the-middle-east-ed9cb441>

See also: <https://www.nytimes.com/article/syria-civil-war-rebels.html>

Syria: Multiple battling factions (Dec 2024)



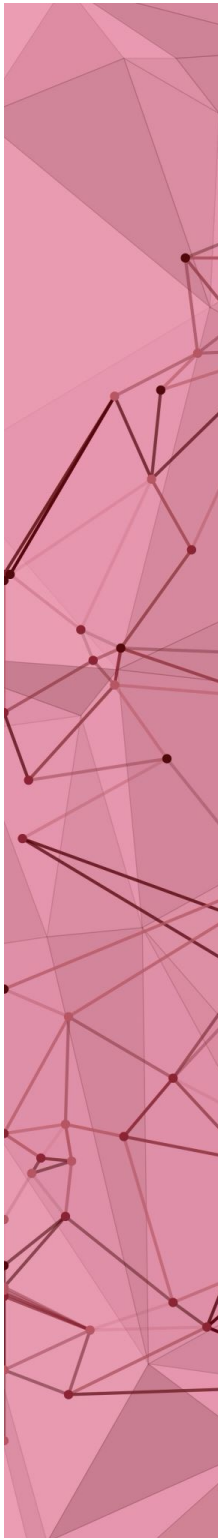
Balanced triangle (January 2022)



Def. Balanced complete graph

A complete graph is balanced
if and only if
every triangle in it is balanced

- Example
- Isn't this definition a little extreme?



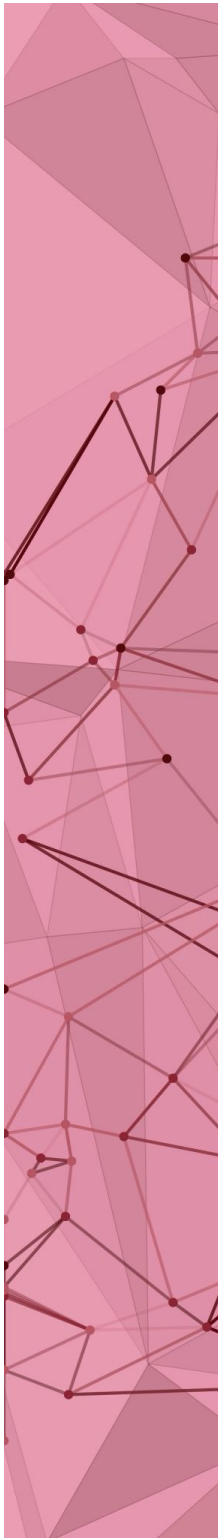
Characterization of balanced complete graphs

Balance Theorem (Harary, 1953)

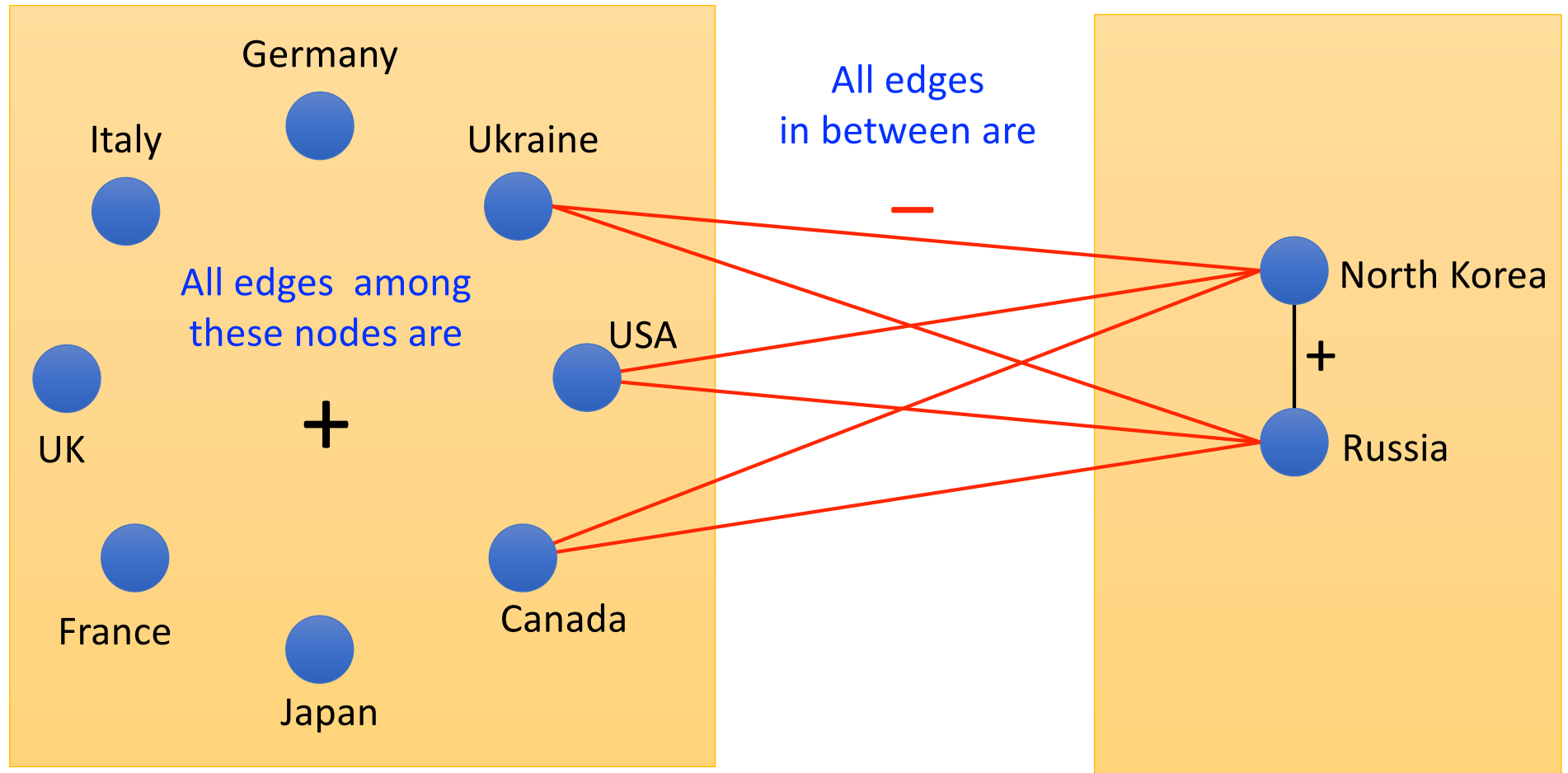
A labeled complete graph is balanced
if and only if
either (i) all edges are + or
(ii) graph consists of two battling factions

Local (triangle) →

Global (either everyone gets along or there are two
battling factions)



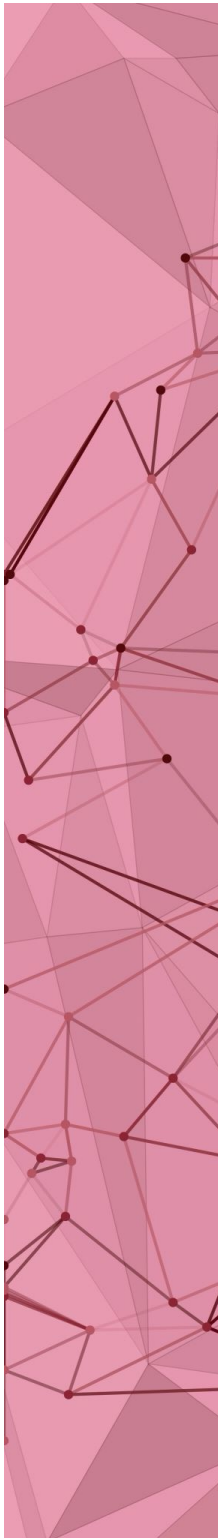
Example (January 2022)



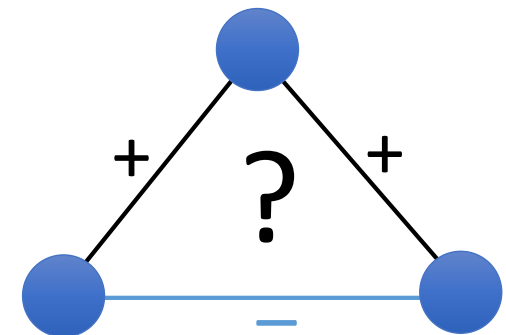
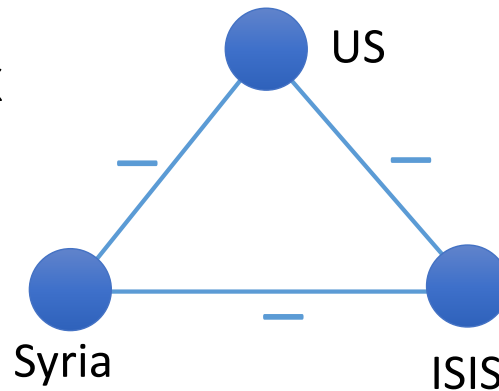
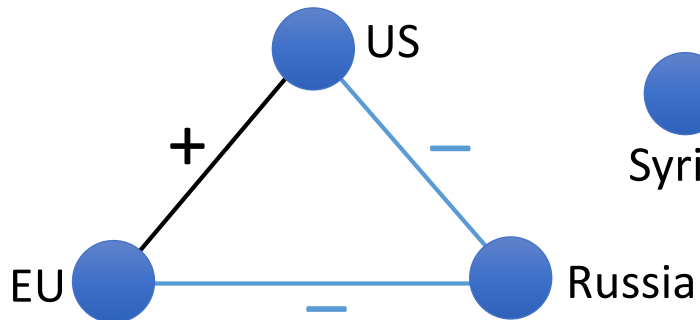
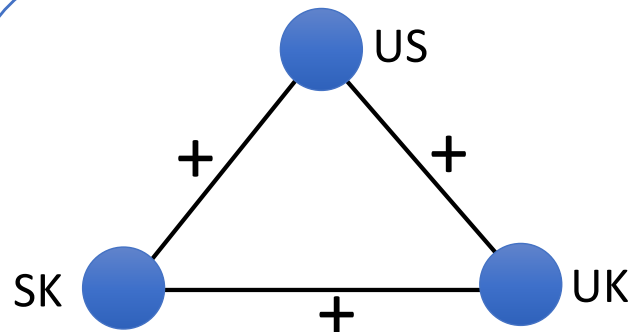
Extensions

Ext. #1: Weakly balanced complete graphs

Ext. #2: Signed graphs (possibly non-complete)



Def. Weakly balanced triangle



Balanced

Unbalanced

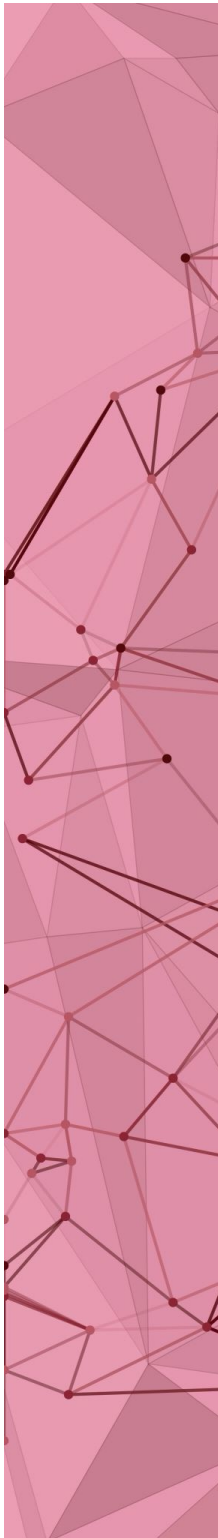
Ext. #1: Def. Weakly balanced complete graph

Every triangle in it is balanced



Characterization of weak balance

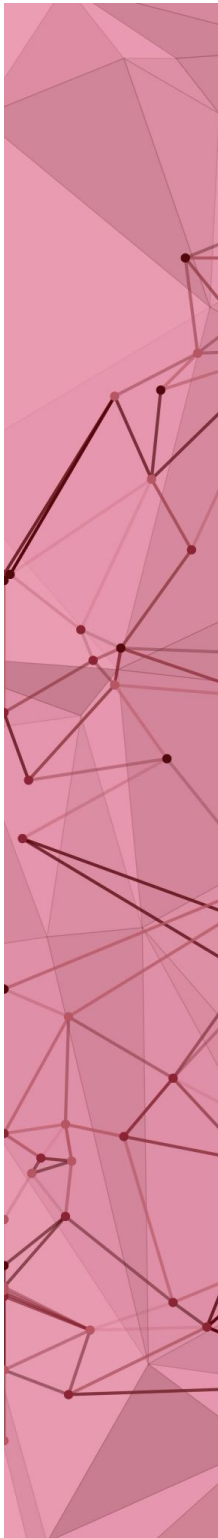
- Complete graph is weakly balanced
if and only if
either (i) all edges + or
(ii) multiple battling factions
- Proof. Very similar to the previous
characterization (with strict def. of balance)



Ext.# 2: General graphs

AKA **signed graph**:

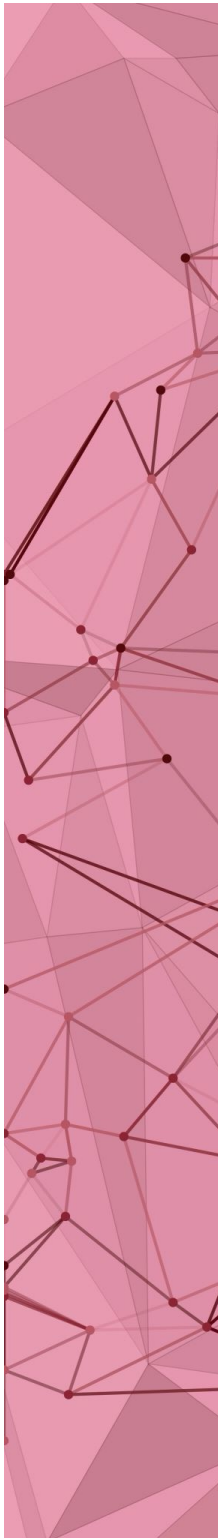
may **not** be a complete graph!



Def: Balanced signed graph

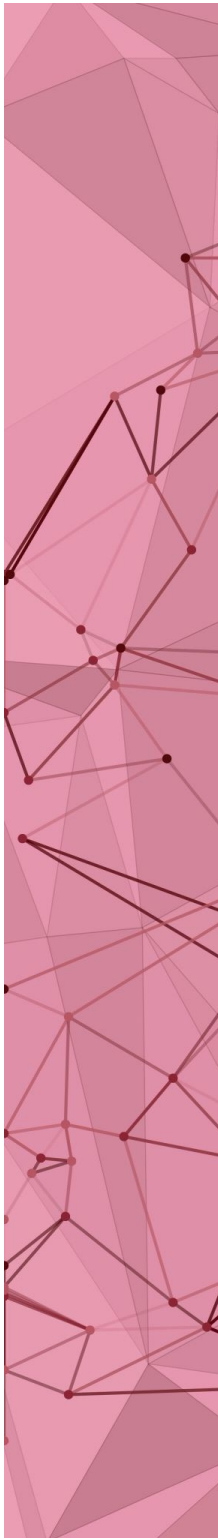
A general signed graph is called balanced if it consists of two battling factions

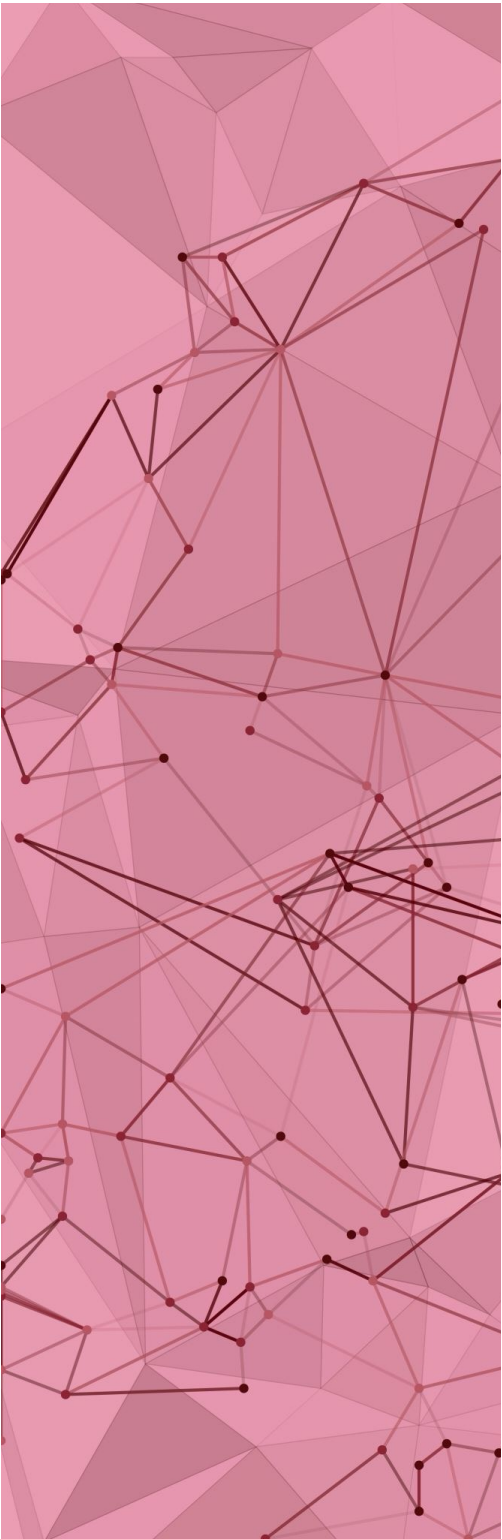
- Definition is not based on triangles (why?)



Characterization: balanced signed graph

- A signed graph is balanced if and only if there exists no cycle with odd number of negative edges
- Q. What goes wrong otherwise?





Real-world examples

Liberation of Bangladesh in 1971

(Michael Moore, Journal of Social Psychology, 1978)

